Metis
System Description

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Talk Plan

1. Example Higher Order Logic Goal

2. Comparison with MESON_TAC
Consider the following HOL subgoal:

...  
1 subgoal:  
> val it =  
  (!P. (!n. (!m. m < n ==> P m) ==> P n) ==> !n. P n) ==>  
  !P. P 0 /\
  (!n. P n ==> P (SUC n)) ==> !n. P n  
: goalstack

- ???
First, identify relevant lemmas:

... 
1 subgoal: 
> val it = 
   (!P. (!n. (!m. m < n ==> P m) ==> P n) ==> !n. P n) ==> !n. P n 
: goalstack

- [LESS_SUC_REFL, num_CASES];

> val it = 
   [ |- !n. n < SUC n, 
      |- !m. m = 0 \/ ?n. m = SUC n] 
: thm list

- ???
Proof 1: The HOL guru way.

... 
1 subgoal:
> val it = 
   (!P. (!n. (!m. m < n ==> P m) ==> P n) ==> !n. P n) ==> 
   !P. P 0 /\ (!n. P n ==> P (SUC n)) ==> !n. P n 
: goalstack

- e (DISCH_THEN (fn t => NTAC 2 STRIP_TAC THEN MP_TAC (Q.ID_SPEC t)) 
    THEN DISCH_THEN MATCH_MP_TAC 
    THEN (Cases THEN1 ASM_REWRITE_TAC [])) 
  THEN DISCH_THEN (MP_TAC o Q.SPEC 'n') 
  THEN ASM_REWRITE_TAC [LESS_SUC_REFL]);

OK..

Goal proved.
|- (!P. (!n. (!m. m < n ==> P m) ==> P n) ==> !n. P n) ==> 
   !P. P 0 /\ (!n. P n ==> P (SUC n)) ==> !n. P n
Proof 2: A simpler approach.

... 

1 subgoal:
> val it =
   (!P. (!n. (!m. m < n ==> P m) ==> P n) ==> !n. P n) ==> !P. P 0 /
   (!n. P n ==> P (SUC n)) ==> !n. P n

: goalstack

- e (METIS_TAC [LESS_SUC_REFL, num_CASES]);

OK..

metis: m-0-1-2-3-4-5-6r|*|0+7x0+0+0+0+0+0+0+1+3+1+0+0+
0+3+0+2+2+4+2+0+4+1x2+3+#

Goal proved.

| |- (!P. (!n. (!m. m < n ==> P m) ==> P n) ==> !n. P n) ==> !P. P 0 /
   (!n. P n ==> P (SUC n)) ==> !n. P n
Comparison with **MESON_TAC**

Total subgoals: \( 1779 + 2024 = 3803 \)
Proved by **MESON_TAC**: \( 1779 + 2017 = 3796 \)
Proved by **METIS_TAC**: \( 1774 + 2007 = 3781 \)

\[
\begin{align*}
\text{prob}_53(0.02) & \quad \text{prob}_44(0.02) & \quad \text{int}_\text{arith}_139(0.09) \\
\text{DeepSyntax}_47(0.11) & \quad \text{Omega}_13(0.11) & \quad \text{euclid}_8(0.2) \\
\text{measure}_138(0.23) & \quad \text{MachineTransition}_0(0.29) & \quad \text{nc}_6(0.38) \\
\text{prob}_169(0.39) & \quad \text{prob}_170(0.42) & \quad \text{fol}_1(0.8) \\
\text{measure}_86(0.93) & \quad \text{Omega}_71(1.78) & \quad \text{fol}_2(7.63)
\end{align*}
\]

**TIME DIFFERENCE**
Arithmetic mean: 0.30s
Geometric mean: 318%
Example Higher Order Logic Goal
Comparison with MESON_TAC

HOL Evaluation

Joe Hurd
Metis
Use \texttt{METIS\_TAC} in your HOL proofs today!

- Just do \texttt{load "metisLib";} \texttt{open metisLib;} to make \texttt{METIS\_TAC} and \texttt{METIS\_PROVE} available.

- No need to retire \texttt{MESON\_TAC}.
  - Given the fragile nature of first-order provers, it's quite useful to have two complementary tactics.

- Lots of interesting research in the future work...
Future Work

- Specialize first-order prover to create *point tools*:
  - Simple arithmetic reasoning.
  - Support predicate subtyping via always-fire rules.
  - Decision procedure for theories such as finite_map.
- Would really like to store theorems, so the user doesn’t have to find the right lemmas each time.
- Improved treatment of combinators at first-order level (pattern unification?).
- Use the interface to create a new link to ‘industrial strength’ first-order provers.
- More powerful first-order calculus: Knuth-Bendix completion, semantic constraints, etc, etc, . . .